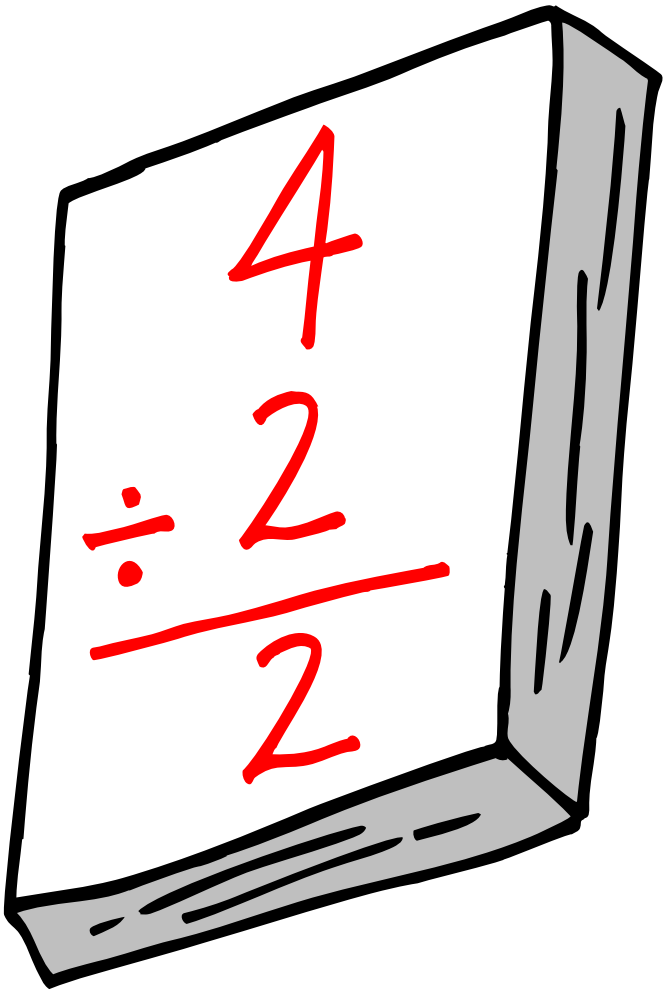
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Regional • 2017



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YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate: $0! + 1 \times 2^3 + 4 - 5 \div (6 - 7)$**

   (A) -15 **(B) -8** **(C) 8** **(D) 12** **(E) 18**

2. **Willie Shair had a bag of marbles. He picked out his 10 favorite ones and put them away in a lock box. He gave 40% of what was left to his best friend. Then he gave $\frac{1}{3}$ of what was left to his brother. He had 6 left to play with. How many marbles were in the bag originally?**

   (A) 32 **(B) 30** **(C) 25** **(D) 20** **(E) 15**

3. **If P is 15% less than Q and Q is 20% more than R, then R is what percent of P? (nearest whole percent)**

   (A) 135% **(B) 105%** **(C) 102%** **(D) 98%** **(E) 95%**

4. **If E= {e, u, c, l, i, d}, V = {v, i, e, t, a}, G = {g, e, r, m, a, i, n} and T = {t, h, e, a, n, o} then $(E \cap T) \cup (G \cap V)$ contains how many distinct elements?**

   (A) 1 **(B) 3** **(C) 4** **(D) 5** **(E) 6**

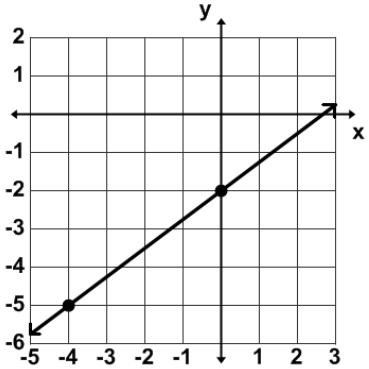
5. **Find the sum of the positive integral divisors of 1,488.**

   (A) 4,464 **(B) 4,188** **(C) 4,096** **(D) 3,968** **(E) 2,976**

6. **Andrew, Linh, and Zach worked a total of 141 problems on their last math test. Zach worked two-thirds of the number of problems Andrew worked and fifteen less problems than Linh did. How many problems did Zach work?**

   (A) 40 **(B) 36** **(C) 33** **(D) 31** **(E) 26**

7. **A line through point P (— 3, 5) and perpendicular to the line shown intersects at point Q (x, y). Find $x + y$.**



   (A) 2.36 **(B) 1.25** **(C) 0.75** **(D) 0.52** **(E) 0.28**

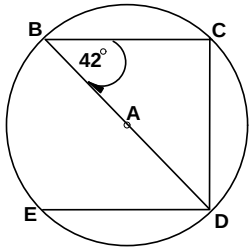
8. **The set of integers {... — 2, — 1, 0, 1, 2, ...} is closed under how many of these operations:**
   $+$ addition — subtraction $\times$ multiplication $\div$ division

   (A) 1 **(B) 2** **(C) 3** **(D) 4** **(E) none of them**

9. If $18x^2 - 3x - 28 = (ax - 4)(bx + 7)$ then $(a + b)(a - b) =$ _____.

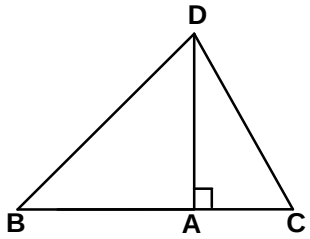
(A) — 27 (B) — 18 (C) — 3 (D) 9 (E) 18

10. Given the circle with center A shown with right angle CDE. Find $m\overset{\frown}{ED}$.



(A) 96° (B) 42° (C) 31° (D) 84° (E) 63°

11. Given △BCD with AD = 6", m∠ABD = 45°, and m∠ADC = 30°. Find the difference in the perimeters of triangles △BAD and △CAD. (nearest tenth)



(A) 7.9" (B) 7.0" (C) 6.6" (D) 5.8" (E) 4.1"

12. ∠A is supplementary to ∠B and ∠B complementary to ∠C. Let m ∠A = 4x — 3 and m ∠B = x + 2. Find m∠C.

(A) 36.2° (B) 38.2° (C) 51.8° (D) 53.8° (E) 57°

13. Poly Gone labeled thirty blank cards with the numbers 1 through 30. After mixing them up, Poly randomly drew out 1 card. What is the probability that the number on the card selected was a triangular number or a pentagonal number?

(A) 50 % (B) $40\frac{1}{3}$ % (C) $36\frac{2}{3}$ % (D) $33\frac{1}{3}$ % (E) 30 %

14. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. If $f_n = 317{,}811$ then n is:

(A) 21 (B) 23 (C) 26 (D) 28 (E) 31

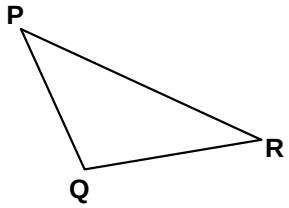
15. Let $A = \begin{bmatrix} 0 & 4 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ -1 & -7 \end{bmatrix}$. If $AB = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$ then $w + x + y + z = ?$

(A) — 14 (B) — 26 (C) — 63 (D) — 82 (E) — 94

16. Find the area of the circle, $x^2 + y^2 - 6x - 10y = 2$. (nearest tenth)

(A) 119.4 units^2 (B) 104.7 units^2 (C) 131.1 units^2 (D) 106.8 units^2 (E) 113.1 units^2

17. Find the area of $\triangle PQR$ if $m\angle PQR = 95^o$, PQ = 16 cm, and QR = 12 cm. (nearest tenth)



(A) 96.0 cm^2 (B) 86.2 cm^2 (C) 95.0 cm^2 (D) 90.9 cm^2 (E) 95.6 cm^2

18. The bearing of the *Coral Princess* from the Panama Canal locks is 120º and the bearing of the *Pride of America* from the *Coral Princess* is 250º. The *Coral Princess* is 5 km from the locks and the *Pride of America* is 6.5 km from the locks . How far is the *Coral Princess* from the *Pride of America*? (nearest tenth)

(A) 2.0 km (B) 5.8 km (C) 7.0 km (D) 8.5 km (E) 10.4 km

19. Which of the following functions are considered to be odd?
I. $f(x) = x|x|$ II. $f(x) = |x| - 2$ III. $f(x) = \frac{x}{1 - x^2}$

(A) I only (B) I & III (C) II only (D) II & III (E) I, II, & III

20. If $a_1 = -5$, $a_2 = -3$, $a_3 = 2$ and $a_n = (a_{n-3}) + (a_{n-2})(a_{n-1})$ for $n \geq 4$, then a_6 equals:

(A) -275 (B) -273 (C) 275 (D) 277 (E) 286

21. Given the function $f(x) = 3x^2 - 4x$, find the slope of the secant line between $x = -1$ and $x = 3$.

(A) 8 (B) 4 (C) 2 (D) -1 (E) -3

22. Which of the following sequences are convergent?
I. $\left\{\frac{1}{\sqrt{n^2+1}-n}\right\}$ II. $\left\{\left(1+\frac{1}{3n}\right)^n\right\}$ III. $\left\{\frac{n+1}{2n-1}\right\}$

(A) I only (B) II only (C) II & III (D) I, II & III (E) none of them

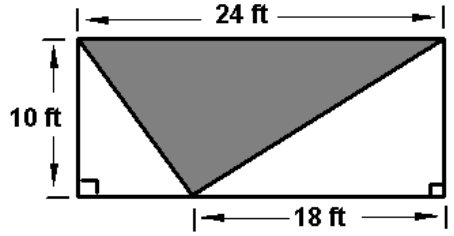
23. The three seniors on Cal Q. Later's four member math team scored 248, 288, and 268 on the UIL District math test. What would the only freshman on the team have to score so that the mean team score would be 252 and the median team score would be 258?

(A) 204 (B) 208 (C) 218 (D) 228 (E) 238

24. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $f_{20} + f_{21}$.

(A) 17,711 (B) 10,946 (C) 14,329 (D) 8,856 (E) 267,914,296

25. Doug Ittup lost his cell phone while tilling his garden. What is the probability that he lost it in shaded region shown? (nearest whole percent)



(A) 50% (B) 61% (C) 55% (D) 48% (E) 40%

26. Which of the following mathematicians was know for their contributions to abstract algebra and the development of the theories of rings, fields, and algebras?

(A) Karen Smith (B) Hypatia (C) Ada Byron (D) Alicia Stott (E) Emmy Noether

27. Noah Pennies has 34 coins consisting of nickels, dimes, and quarters worth $5.50. There are 6 more dimes than nickels. How many quarters does Noah have?

(A) 8 (B) 10 (C) 12 (D) 14 (E) 16

28. Mary Goround is putting lantern lights around her circular swimming pool. She wants to put a light every 4 feet. The diameter of the pool 28 feet. What will the cost of the lights be before taxes if the cost of each light is $12.50? (nearest quarter dollar)

(A) $250.00 (B) $262.50 (C) $268.25 (D) $275.00 (E) $281.50

29. The *Tuity Fruity* market sold a box containing 3 lbs of apples and 4 lbs of oranges for $5.15. They sold another box containing 5 lbs of apples and 2 lbs of oranges for $4.85. What would it cost to buy a box containing 7 lbs of apples and 6 lbs of oranges?

(A) $9.25 (B) $9.30 (C) $9.35 (D) $9.50 (E) $9.55

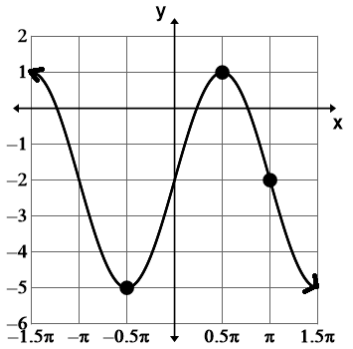
30. The probability that an applicant for a high-paying job has a college degree is 85%, and the probability that the applicant has experience in the field is 30%. If 25% of those with college degrees have experience, what is the probability that an applicant with experience has a college degree? (nearest degree)

(A) 60% (B) 64% (C) 67% (D) 71% (E) 79%

31. If the roots of $x^3 + cx^2 + dx - 28 = 0$ are 1, 4, and k, then $c + d$ equals:

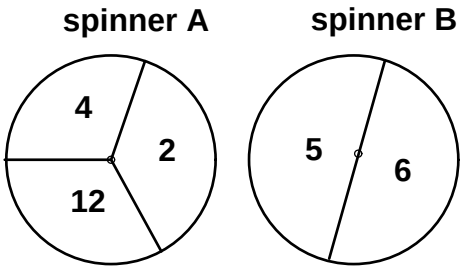
(A) 51 (B) 27 (C) 1 (D) 23 (E) 55

32. The equation y = _____ will produce this graph.



(A) 3cos(x — 2) (B) 3sin(x) — 2 (C) 3sin(x— 2) — 2 (D) 3cos(x) — 2 (E) 3sin(2x) — 2

33. Spinner A is divided into three equal sectors and spinner B into two halves. Betty Wont spins each spinner once. If A is bigger than B then Betty receives the sum of the A and B dollars. If A is smaller than B then Betty loses the sum of the A and B dollars. What is the mathematical expectation of playing the game many times?



(A) — $0.16 (B) + $0.17 (C) + $0.33 (D) + $0.84 (E) + $1.00

34. Point P (— 4, — 1) is the midpoint of the line segment with endpoints Q (x, 6) and R (— 11, y). Find x + y.

(A) — 15 (B) — 8 (C) — 5 (D) 3 (E) 5

35. Which of the following is an identity for $(\csc^2\theta)(\tan^2\theta) - 1$?

(A) $\tan\theta$ (B) $\cot\theta$ (C) $\cot^2\theta$ (D) $\sec^2\theta$ (E) $\tan^2\theta$

36. The function *f* is defined by *f*(x) = — 6 — ln(4 — x). The inverse function of *f* is $f^{-1}(x)$ = ?

(A) $(6 + \ln(4 - x))^{-1}$ (B) $e^{(6+x)} - 4$ (C) $4 + (\ln(6 + x))^{-1}$

(D) $e^{(x-2)}$ (E) $4 - e^{(-x-6)}$

37. The vertical asymptote and the oblique asymptote of $f(x) = \frac{2x^2 + 3x + 5}{x - 1}$ intersect at point (x , y). Find the value of y.

(A) 10 (B) 7 (C) 4 (D) 3 (E) 1

38. In the expansion of $(3x - 2y)^6$, the sum of the coefficients of the 2nd term, the 4th term, and the 6th term is:

(A) $-7{,}812$ **(B)** $-6{,}660$ **(C)** -192 **(D)** 720 **(E)** 32

39. Which of the following numbers is considered to be an evil and extravagant Lucas number?

(A) 76 **(B)** 46 **(C)** 18 **(D)** 4 **(E)** 123

40. $11011_2 + 1101_4 + 110_8 =$ _____ 16

(A) 114 **(B)** 180 **(C)** 16A28 **(D)** 10 **(E)** B4

41. Let f(x) = 2 — x, g(x) = 2x + 3, h(x) = 3x — 1, and $f\left(2g\left(3h(x)\right)\right) = 4$. Find x.

(A) $\frac{1}{9}$ **(B)** $\frac{35}{36}$ **(C)** $\frac{1}{18}$ **(D)** $-\frac{4}{9}$ **(E)** $-1\frac{7}{36}$

42. If $\sqrt{x^2\left(\sqrt{x^3\left(\sqrt{x^5}\right)}\right)} = \sqrt[n]{x^k}$, where k and n are relatively prime and $x > 1$, then k = ?

(A) 30 **(B)** 19 **(C)** 8 **(D)** 10 **(E)** 5

43. Find the slope of the line perpendicular to the curve $y = 2x^3 - 3x + 5$ at (— 1, 6).

(A) 3 **(B)** $\frac{1}{12}$ **(C)** $-\frac{1}{6}$ **(D)** $-\frac{1}{3}$ **(E)** $-\frac{1}{2}$

44. Let $f(x) = \frac{-5x+1}{x-1}$, where $x \neq 1$. Find $f^{-1}(x)$.

(A) $\frac{x+1}{x+5}$ **(B)** $\frac{x-1}{1+5x}$ **(C)** $\frac{1-x}{1-5x}$ **(D)** $\frac{x-5}{x-1}$ **(E)** $\frac{1-x}{x+5}$

45. Omit question #45. $^{2}\!/_{2}$,

(A) 24 **(B)** $27\frac{1}{3}$ **(C)** $30\frac{2}{3}$ **(D)** 34 **(E)** $37\frac{1}{3}$

46. The sequence 8, p, q, r, $\frac{8}{9}$ is a harmonic progression. Find the value of p + q + r. (nearest tenth)

(A) 5 **(B)** 5.4 **(C)** 6 **(D)** 6.3 **(E)** 6.7

47. Point P(2, 2) undergoes several transformations to point Q(x1, y1). First, it is rotated 90° clockwise about the origin. Then, it is reflected across the y-axis. Then it is translated 3 units horizontally in the positive direction and vertically 2 units in the negative direction. Find x1 + y1.

(A) — 3 **(B)** — 1 **(C)** 0 **(D)** 1 **(E)** 2

48. The numbers greater than 1 are arranged in the array below. In which column will 408 fall.

(A)	(B)	(C)	(D)	(E)
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

49. If $\frac{Ax+B}{5x+1} - \frac{2x+3}{3x-4} = \frac{11x^2-51x+5}{15x^2-17x-4}$, where A and B are constants, then $(A+B)(A-B)$ equals:

(A) 53 (B) 45 (C) 37 (D) 30 (E) 25

50. Let $\triangle PQR$ exist such that $m\angle QPR = 60°$. Point A lies on $\overline{PQ}$ and $\overline{AR}$ is the altitude $\triangle PQR$. Find $m\angle PQR$ if the area of $\triangle AQR$ is 75% of the area of $\triangle APR$. (nearest degree).

(A) 67° (B) 71° (C) 45° (D) 49° (E) 60°

51. Let $f(x) = ax + 8$ and $g(x) = bx - 11$, where a and b are positive integers. Find $a + b$ if $f(g(x)) = g(f(x))$.

(A) 2 (B) 3 (C) 8 (D) 11 (E) 19

52. Nick Ohl put 5 nickels in a bag, shook them up, then poured them on the table. What are the odds of four or more tails being face up?

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{8}$ (D) $\frac{5}{11}$ (E) $\frac{3}{13}$

53. Given the sequence, $\frac{13}{(1\times1+1)} - \frac{13}{(2\times2-1)} + \frac{13}{(3\times3+1)} - \frac{13}{(5\times5-1)} + \frac{13}{(8\times8+1)} - \ldots$, find the digit in the ten-thousandths place.

(A) 9 (B) 8 (C) 6 (D) 3 (E) 0

54. Let $x = 5 + \cfrac{2}{5 + \cfrac{2}{5 + \cfrac{2}{5 + \cfrac{2}{5 + \ldots}}}}$. Find x. (nearest hundredth)

(A) 5.20 (B) 5.25 (C) 5.37 (D) 5.40 (E) 5.42

55. 0.4717171... base 8 can be written as which of the following simplified fractions in base 8?

(A) $\frac{311}{504}_8$ (B) $\frac{65}{77}_8$ (C) $\frac{147}{250}_8$ (D) $\frac{309}{511}_8$ (E) $\frac{103}{170}_8$

56. If the three numbers 155, 227, and 344 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

(A) 7 **(B) 5** **(C) 3** **(D) 2** **(E) 1**

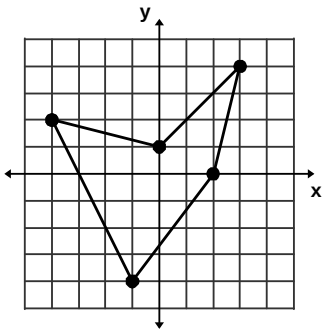
57. Mei Chado's height is 6' 2". She is walking at a rate of 5 ft/sec toward a street light that is 20 feet tall. At what rate is the tip of her shadow moving? (nearest tenth)

(A) 7.8 ft/sec **(B) 7.2 ft/sec** **(C) 6.2 ft/sec** **(D) 2.8 ft/sec** **(E) 2.2 ft/sec**

58. Simplify: $\left(\frac{(n-1)!}{(n-2)!} + \frac{(n+1)!}{n!}\right) \div \frac{(n)!}{(n-1)!}$

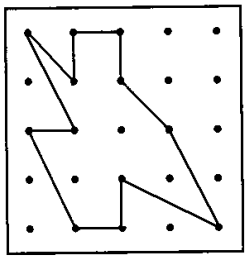
(A) 2 **(B)** $\frac{n-1}{n+1}$ **(C)** $2n^2 - 2n$ **(D)** $\frac{n^2+1}{n-1}$ **(E)** $2n^2$

59. Find the area of the pentagon. The coordinates of the vertices are integers.



(A) 21.5 sq. units **(B) 21 sq. units** **(C) 20.5 sq. units** **(D) 20 sq. units** **(E) 19.5 sq. units**

60. A rubber band was stretched on the geoboard to form this 12-sided figure. What is its area?



(A) 8 sq. units **(B) 8.5 sq. units** **(C) 9 sq. units** **(D) 9.5 sq. units** **(E) 10 sq. units**

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2017
Answer Key

1. E	**21. C**	**41. A**
2. C	**22. C**	**42. B**
3. D	**23. A**	**43. D**
4. B	**24. A**	**44. A**
5. D	**25. A**	**45. E**
6. B	**26. E**	**46. B**
7. D	**27. E**	**47. A**
8. C	**28. D**	**48. B**
9. A	**29. C**	**49. B**
10. A	**30. D**	**50. A**
11. E	**31. B**	**51. A**
12. C	**32. B**	**52. E**
13. D	**33. B**	**53. B**
14. D	**34. C**	**54. C**
15. D	**35. E**	**55. C**
16. E	**36. E**	**56. D**
17. E	**37. B**	**57. B**
18. D	**38. A**	**58. A**
19. B	**39. C**	**59. C**
20. D	**40. E**	**60. A**