

UIL Mathematics
Seven Trapezoidal Means

1. Arithmetic mean -- The usual "average" of the numbers. For 2 numbers a and b , the arithmetic mean A is half the sum, $A = \frac{a+b}{2}$. This can be extended to more than 2 numbers,

$$A = \frac{a_1 + a_2 + \dots + a_n}{n}.$$

2. Geometric mean -- For 2 numbers, the geometric mean is the square root of the product, $G = \sqrt{ab}$. Extending this to more than 2 numbers, $G = \sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n}$.

3. Harmonic mean -- Harmonic mean is a way of averaging rates. If you drive one way at 40 mi/hr and drive 60 mi/hr for the return trip, your average speed for the entire trip is not the expected 50 mi/hr but rather 48 mi/hr. For 2 numbers, the harmonic mean H is twice the product divided by the sum, $H = \frac{2ab}{a+b}$. For more than 2 numbers, $H = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}}$.

4. Heronian mean -- Used to find the volume of the frustrum of a pyramid or a cone.

$H = \frac{1}{3}(A + \sqrt{AB} + B)$. Notice that this is one-third of the geometric mean plus two-thirds of the arithmetic mean.

5. Contraharmonic mean -- $C = \frac{\text{Arithmetic mean of the squares}}{\text{Arithmetic mean of the numbers}} = \frac{a_1^2 + a_2^2 + \dots + a_n^2}{a_1 + a_2 + \dots + a_n}$. For 2 numbers,

$$C = \frac{a^2 + b^2}{a + b}.$$

6. Root mean square -- For 2 numbers, $R = \frac{1}{2}\sqrt{a_1^2 + a_2^2}$. For more than 2 numbers,

$$R = \frac{1}{n}\sqrt{a_1^2 + a_2^2 + \dots + a_n^2}.$$

7. Centroidal mean -- Used to find the length of the segment parallel to the bases of a trapezoid and passing through the centroid. For 2 numbers, $C = \frac{2(a^2 + ab + b^2)}{3(a+b)}$.