Algebra II Pre-AP

Solving Non-Linear Systems of Equations

In Algebra I, you learned to solve systems of *linear* equations such as $\begin{cases} 2x - 3y = 5 \\ x + y = 17 \end{cases}$ by using elimination and substitution and also by using a graphing calculator to find the points of intersection. Today you'll be using those same techniques to solve systems of equations whose graphs are not necessarily lines.

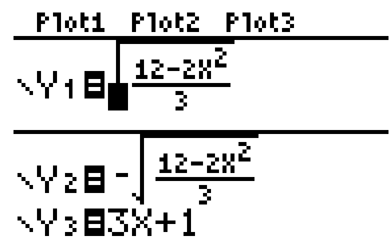
First, let's do one using the calculator.

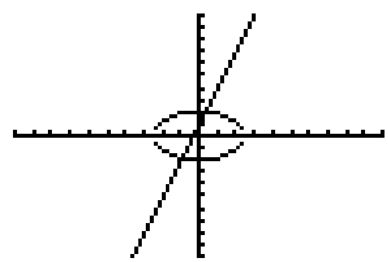
Example 1: Use a graphing calculator to solve $\begin{cases} \dfrac{x^2}{6} + \dfrac{y^2}{4} = 1 \\ 3x - y = -1 \end{cases}$.

Solution: To enter the equations into a calculator, you must first solve each equation for y. I would advise starting by clearing the fractions from the top equation.

$$12\left(\frac{x^2}{6} + \frac{y^2}{4}\right) = 12(1)$$

$$2x^2 + 3y^2 = 12$$
$$3y^2 = 12 - 2x^2$$
$$y^2 = \frac{12 - 2x^2}{3}$$
$$y = \pm\sqrt{\frac{12 - 2x^2}{3}}$$





The bottom equation is equivalent to $y = 3x + 1$.

Now enter the equations into the Y= menu as shown in the top picture to the right.

Now graph, using a standard window. Your graph should look like the second picture.

Notice that there are 2 points of intersection. One is the intersection of the line (Y3 on your calculator) and the *top* half of the ellipse (Y1 on your calculator) and the other is the intersection of the line and the *bottom* half of the ellipse (Y2 on your calculator.)

To find the point of intersection that is in Quadrant I, start by pressing 2nd, Trace, and then 5 to use the Intersect function. Notice that the cursor is on Y1 (the text in the upper left-hand corner of the screen tells you which curve the cursor is on), so press Enter to accept this as the first curve. The cursor will then jump down to Y2, but that is not the second curve that we need. Pressing the Down arrow will move the cursor to Y3, which is what we want, so press Enter to accept that choice. For the "Guess" you can just press Enter again since Y1 and Y3 only intersect at one point. To 3-decimal place accuracy, the point of intersection is $(0.327, 1.982)$.

To find the point of intersection that is in Quadrant II, call up the Intersect function again, then find the intersection of Y2 and Y3. To 3-decimal place accuracy, you should get $(-0.948, -1.844)$.

Next let's look at solving a system without having to use a graphing calculator.

Example 2: Without using a calculator, solve $\begin{cases} x^2 + 2y^2 = 23 \\ 2x^2 - y^2 = 1 \end{cases}$.

Method #1 – Elimination: Notice that by multiplying the bottom equation by 2 and adding the equations together, we can eliminate the y and solve for the x values.

$$x^2 + 2y^2 = 23$$
$$4x^2 - 2y^2 = 2$$

Adding the equations together gives $5x^2 = 25 \Rightarrow x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

Since, for both possible values of x, $x^2 = 5$, we can replace x^2 in either equation with 5 and solve for y. Confirm that this gives $y = \pm 3$.

Therefore, this system has FOUR solutions, and the solution set is $\{(\sqrt{5},3),(\sqrt{5},-3),(-\sqrt{5},3),(-\sqrt{5}-3)\}$. On solutions sets like this, please do NOT use the "$\pm$" notation. Write out all solutions individually for clarity.

Method #2 – Substitution: Although solving either equation for either variable would be messy (because doing so would involve square roots, we can easily solve the top equation for x^2 and then substitute into the bottom equation.

$$x^2 = (23 - 2y^2) \text{ so } 2(23 - 2y^2) - y^2 = 1$$
$$46 - 4y^2 - y^2 = 1$$
$$5y^2 = 45$$
$$y^2 = 9 \Rightarrow y = \pm 3$$

Like we did when we finished this example using elimination, you can now replace y^2 with 9 in either equation to get that $x = \pm\sqrt{5}$, resulting in the same 4 solutions as before.

For problems that you solve without using a calculator, you will generally be able to use whichever method you prefer (substitution or elimination.)